Definitions:

Stationary independent
As long as the duration are the same they have the same pdf
Arrival, or counting Process
(a) . X(0) = 0 with probability of 1
(b) If $t_1 < t_2$ then X(t_1) < X(t_2)
(c) X(t) takes non-negative integers.
(d) If at time t_0 there exists a jump in X(t), the value of this jump is alway 1
(e) X(t)$\geq$ n = $T_n \leq t$
Cyclostationary
$m_X(t + T_0) = m_X(t)$
$R_X(t_1 + T_0, t_2 + T_0) = R_X(t_1, t_2)$

Complext RV
if $Z = X + jY$ can be treated as a random vector with
$E[Z] = E[X] + jE[Y] \ and \ Var[Z] = E[|Z|^2] - |E[Z]|^2 = var(X) + Var(Y)$

$R_X(t_1, t_2)$, **Auto correlation**
definition: $R_X(\tau)_T = \frac{1}{T}\int_{-\infty}^{\infty} X_T(t+\tau)X_T(t)\ dt$
. $R_X(t_1, t_2) = E[X(t_1)X^*(t_2)]$
if its WSS
$R_X(t_1, t_2) = R_X(t_1 - t_2) = R_X(\tau)$
$R_X(-\tau) = R_X^*(\tau)$
$R_X(0) = E[|X(t)|^2]$ = power of the process
$|R_X(\tau)| \leq R_X(0)$

$R_{XY}(t_1, t_2)$, **Cross correlation**
definition: $R_{XY}(t_1, t_2) = E[X(t_1)Y^*(t_2)]$
. $R_{XY}(t_1, t_2) = R_{YX}^*(t_2, t_1)$
$|R_{XY}(\tau)| \leq \sqrt{R_X(0)R_Y(0)}$

K(t_1, t_2), autocovariance
$C_X(t_1, t_2) = R_X(t_1, t_2) - m_X(t_1)m_X^*(t_2)$

Poisson Process
Poisson process is **stationary independent and Independent increment** RP
$m_X(t) = \lambda t$
$R_X(t_1, t_2) = \lambda^2 2t_1t_2 + \lambda min(t_1, t_2)$
$C_X(t_1, t_2) = \lambda min(t_1, t_2)$
The nth arrival time is a Erland distribution

Wiener process
$m_x = 0,\ Var(X) = \alpha t\ ,\ R_X(t_1, t_2) = \alpha\ min(t_1, t_2)$
if x is gaussian white noise and W is wiener process then:

$$W = \int_0^\tau X(t) \qquad Var(X) = \alpha\tau \qquad E[X] = 0$$

$$Y(t) = \frac{1}{2T}\int_{t-T}^{t+T} X(\mu)\,du \qquad X(\tau)*h^*(\tau) = Y(\tau) \qquad X(\tau)*\frac{1}{2T}\sqcap(\frac{\tau}{2T}) = Y(\tau)$$

Joint Wide sense stationary

if Z(t) = aX(t) + bY(t) is JWSS then

$$R_Z(\tau) = |a|^2 R_X(\tau) + |b|^2 R_Y(\tau) + ab^* R_{XY}(\tau) + ba^* R_{YX}(\tau)$$

$$S_Z(\tau) = |a|^2 S_X(\tau) + |b|^2 S_Y(\tau) + 2Re[ab^* S_{XY}(f)]$$

if Z(t) = X(t) + jY(t) is JWSS then

$$R_Z(\tau) = R_X(\tau) + R_Y(\tau) + j(R_{YX}(\tau) + R_{XY}(\tau))$$

$$S_Z(\tau) = S_X(\tau) + S_Y(\tau) + 2Im[S_{XY}(f)]$$

LTI systems

a) $m_Y(t) = m_X(t) * h(t)$
b) $R_{XY}(t_1, t_2) = R_X(t_1, t_2) * h^*(t_2)$
c) $R_Y(t_1, t_2) = R_X(t_1, t_2) * h(t_1) * h^*(t_2)$

if it is WSS
a) $m_Y(\tau) = m_X(\tau)H(0)$
b) $R_{XY}(\tau) = R_X(\tau) * h^*(-\tau)$
c) $R_Y(\tau) = R_X(\tau) * h(\tau) * h^*(-\tau)$

for PSD
$m_Y = m_X H(0)$
$S_{XY}(f) = S_X(f)H^*(f)$
$S_Y(f) = S_X(f)|H(f)|^2$

Ergodic process

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-2T}^{2T} C_X(\tau)\left(1 - \frac{|\tau|}{2T}\right)\,d\tau$$

Karhunen-Loeve

COV$[X_n, X_m]$ = λ_n if n=m

$$C_X(t_1, t_2) = \sum_{n=1}^{\infty} \lambda_n \phi_n(t_1)\phi_n(t_2)$$

$$X_n = \int_a^b X(t)\phi_n^*(t)\,dt$$

$$\int_a^b C_X(t_1, t_2)\phi_n(t_2)\,dt_2 = \lambda_n \phi_n(t_1)$$

for Wiener process
$\lambda\phi(t_1 = \int_0^T \sigma^2 min(t_1, t_2)\phi(t_2)\,dt_2$
$= \int_0^{t_1} \sigma^2 t_2 \phi(t_2)\,dt_2 + \int_{t_1}^T \sigma^2 t_1 \phi(t_2)\,dt_2$